Middleforth Church of England Primary School

Maths Calculation Policy	
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Date	October 2023
Review Date	October 2024

Let Your Light Shine – Matthew 5:16

Through the Maths Calculation Policy, the school will promote and teach the values we learn based on the example of the Christian faith:

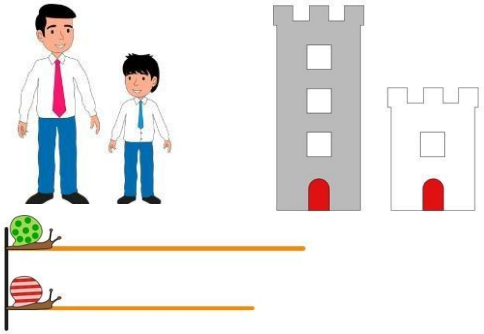
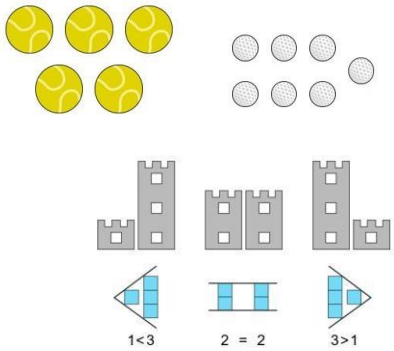
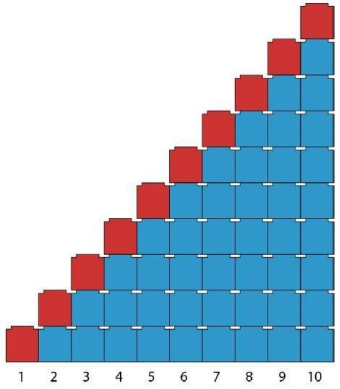
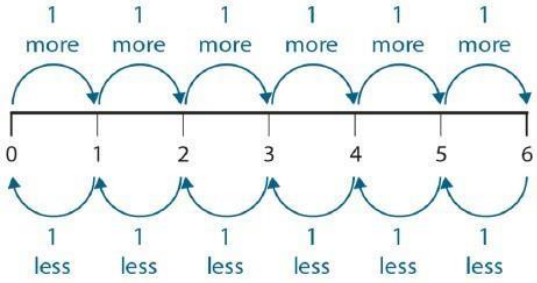
- Forgiveness
- Respect for self and others
- Reconciliation and redemption
- Truth and honesty
- Trust and fairness
- Tolerance and compassion
- Self-discipline
- Respect for property and the environment
- Politeness

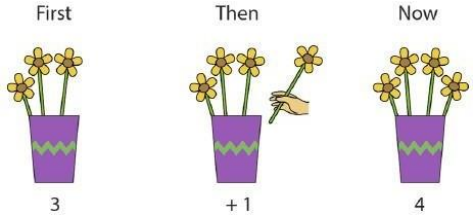
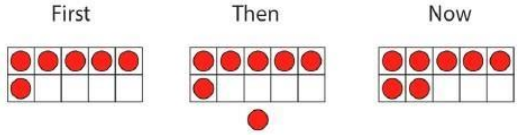
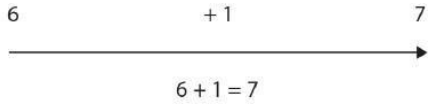
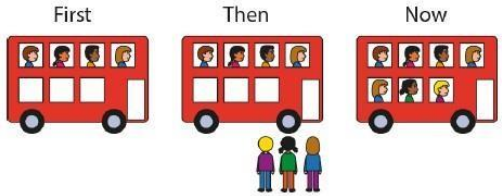
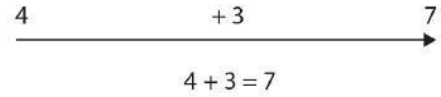
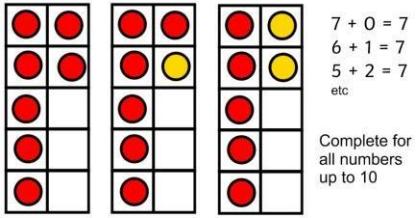
Such values, in turn, promote not only the Christian ethos and aims of Middleforth Church of England Primary School, but assist in the preparation of the children for the responsibilities and duties of adult life.

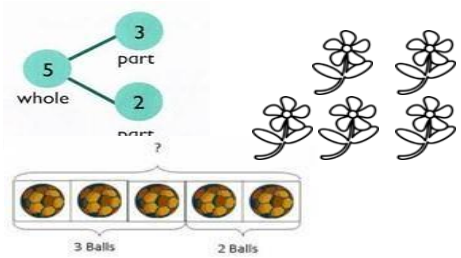
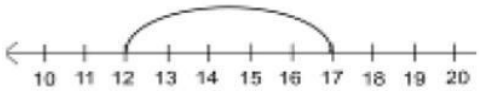
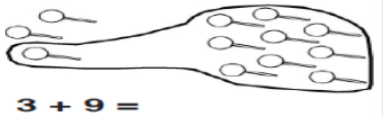
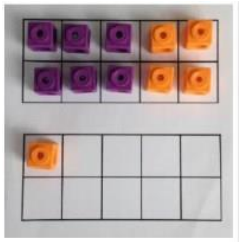
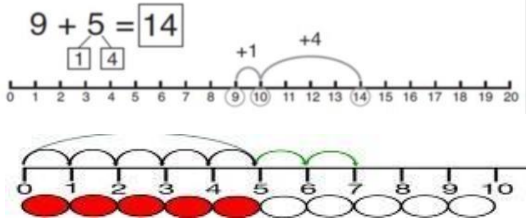
Vision

As a caring, Christian community, we aspire to 'let our light shine'. We will open up the world to celebrate God's wonderful creation and foster a sense of awe and wonder.
We will nurture our God given talents to ensure that everyone reaches their full potential academically, socially and spiritually.
'Let your light shine Matthew 5.16'

Year 1 Addition

Objective, Strategy **Key Vocabulary**	**Concrete**	**Pictorial**	**Abstract**
Comparing Objects, groups of objects Length, weight, mass, heavier, lighter, same, equal	People's height, distance, mass. Use of pan balances using Numicon or similar to show equivalence, < > Comparing multiple objects Use of concrete materials eg. Compare bears, jewels, cubes etc to create groups of different sizes to compare		
Using < > and = Fewer, more, less than, more than, equal to, fewer than	Use a multilink staircase in two colours		Use variation with missing boxes and missing symbols. 3 ◯ 4 4 > □ 2 ◯ 2 □ < 6
Finding one more, finding one less			One more/less sentences – example one: 1 more than 3 is □ 1 less than 2 is □ 1 more than □ is 1 1 less than □ is 1

Objective, Strategy & Key Vocabulary	Concrete	Pictorial	Abstract
Adding 1 gives 1 more			
Augmentation— increasing an amount	Use FIRST, THEN, NOW and range of practical situations for showing augmentation. E.g. first there were three chn on carpet then 2 more came. Now there are 5 chn on the carpet.		
Stories of numbers within 10	Children should work with doubled sided counters and ten frame. Start with 7 red, turn one over, tell me the 'story'? Turn one more over. What is the 'story'? Continue. Complete this for stories of all numbers up to 10.		7 + 0 = 7 6 + 1 = 7 5 + 2 = 7 4 + 3 = 7 3 + 4 = 7 2 + 5 = 7 1 + 6 = 7 0 + 7 = 7

Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	 Use part part whole model.   Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.  	 4 + 3 = 7 10= 6 + 4 Use the part whole diagram as shown above to move into the abstract.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11  2 more than 5.	 Start at the larger number on the number line and count on in ones or in one jump to find the answer.  3 + 9 =	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 Start with the bigger number and use the smaller number to make 10. Use ten frame 	Use pictures or a number line. Regroup or partition the smaller number using the part whole model to make 10. 	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.' '8 is 3 more than 5.'*

Adding 1 and 2 | Bonds to 10 | Adding 10 | Bridging/ compensating

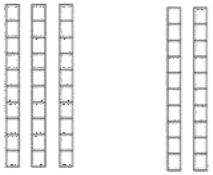
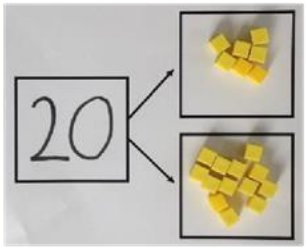
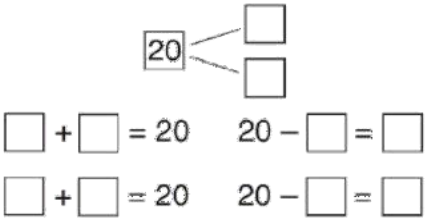
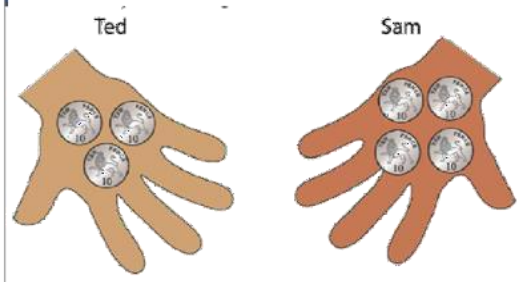
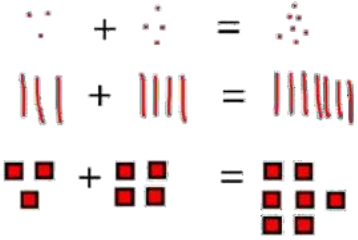
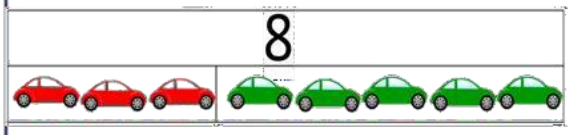
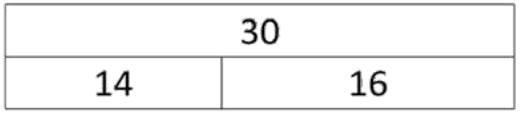
Doubles | Adding 0 | Near doubles

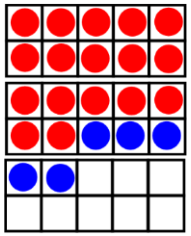
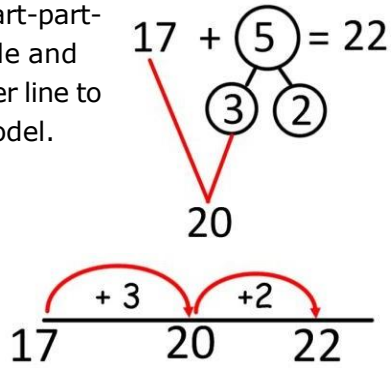
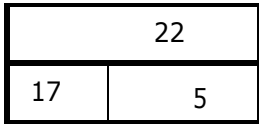
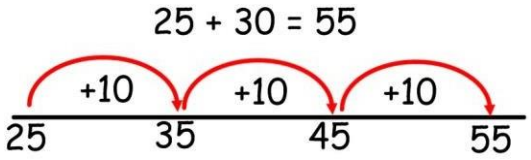
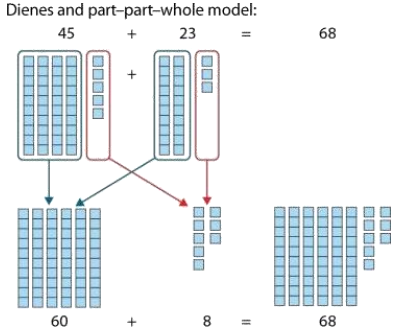
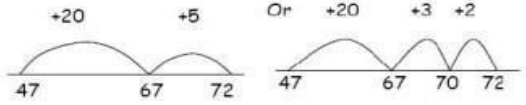
Y1 facts

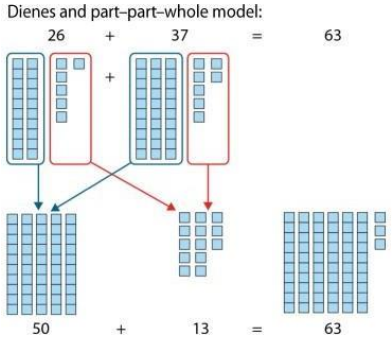
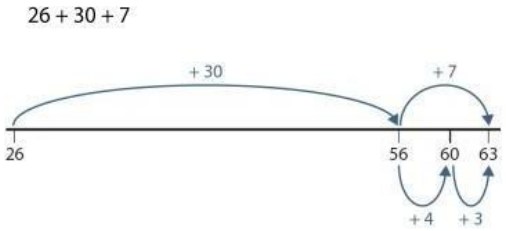
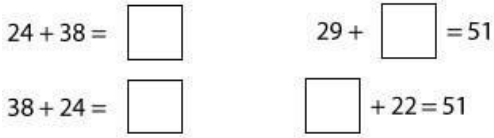
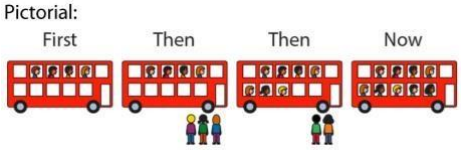
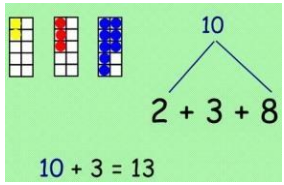
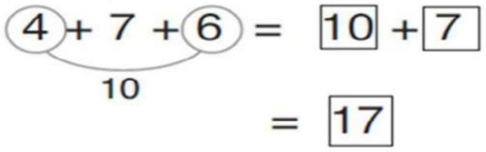
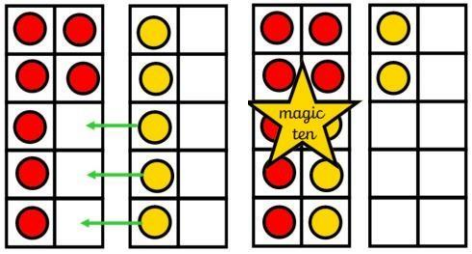
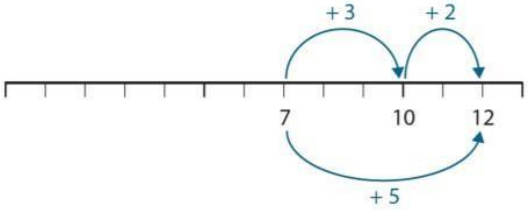
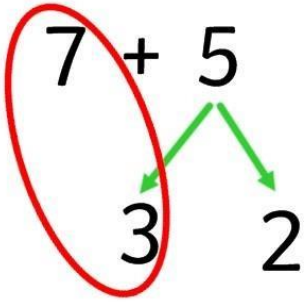
Y2 facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

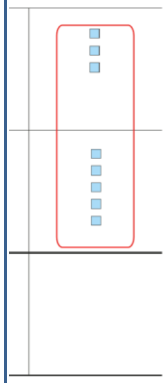
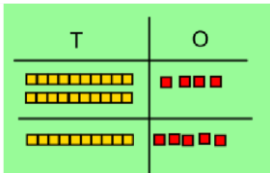
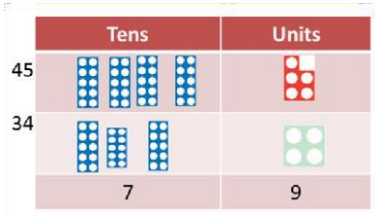
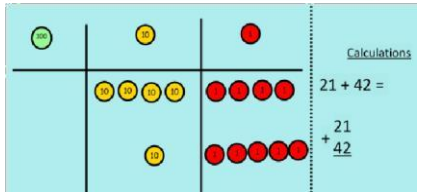
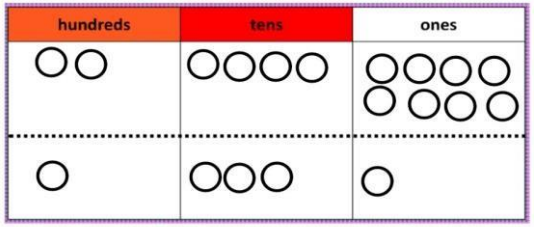
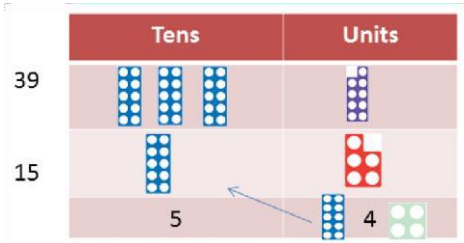
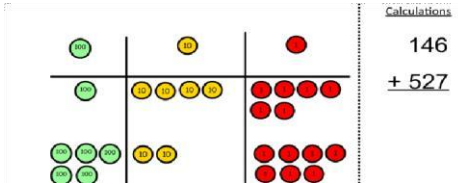
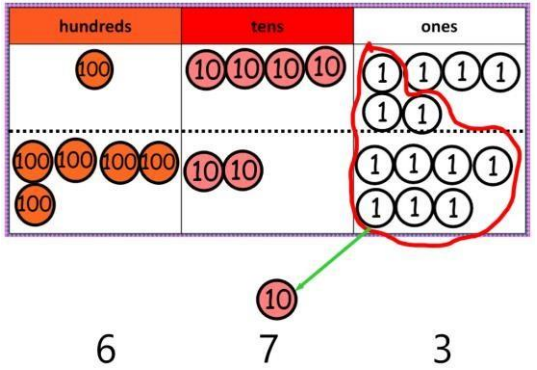
Year 2 Addition

Objective & Strategy & Key Vocabulary	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 + 20  Model using dienes and bead strings	 __ tens and ___ tens makes __ tens Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60 □ + 30 = 50
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 20 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 - 1 = □ 1 + □ = 16 16 - □ = 1
Using known facts	 Ted Sam	 Children draw representations of H,T and O	3 + 4 = 7 Leads to 30 + 40 = 70 Leads to 300 + 400 + 700 *'3 things and 4 things is always 7 things'*
Bar model	 3 + 4 = 7	 3 + 5 = 8	 14 + 16 = 30

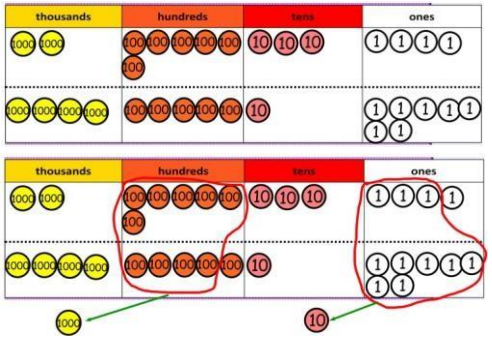
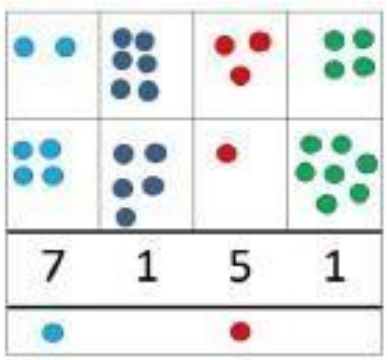
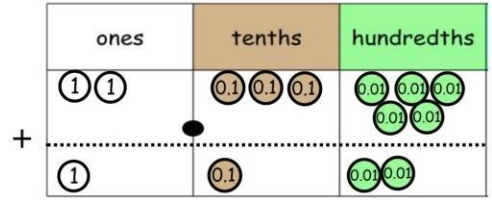
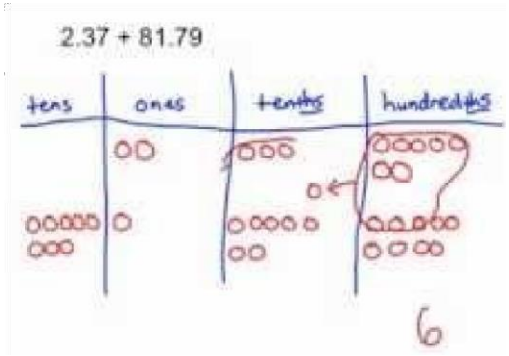
Objective, Strategy **Key Vocabulary**	**Concrete**	**Pictorial**	**Abstract**
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part-part-whole and number line to model. 	17 + 5 = 22  Explore related facts 17 + 5 = 22 22 = 17 + 5 5 + 17 = 22 22 = 5 + 17 22—17 = 5 17 = 22 - 5 22—5 = 17 5 = 22 - 17
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57 □ + 30 = 67
Add two 2-digit numbers without bridging. 'Friendly numbers'	Model using dienes , place value counters and numicon 	 Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72

Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
Add any two 2-digit numbers			
Add three 1-digit numbers	 Combine to make magic 10 first where relevant, or bridge 10 then add third	Use language of fist, then, then, now  Use part part whole to show magic ten 	 Combine the two numbers that make/ bridge ten then add on the third.
Adding two numbers that bridge 10.	 Use double sided counters and ten frames. Move counters to fill the ten frame and make Magic 10	 Show on a number line how 5 is portioned into adding three, then adding 2.	

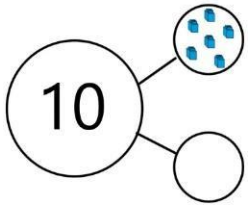
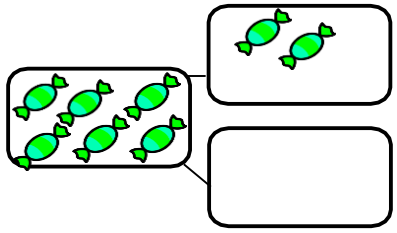
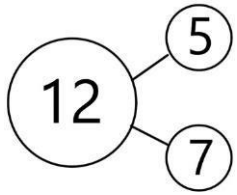
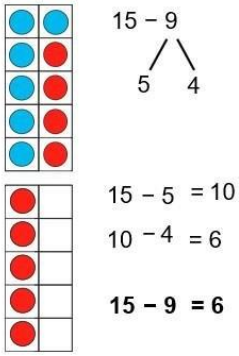
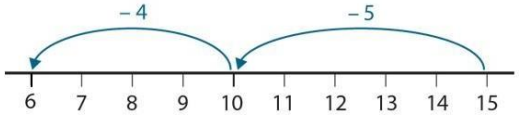
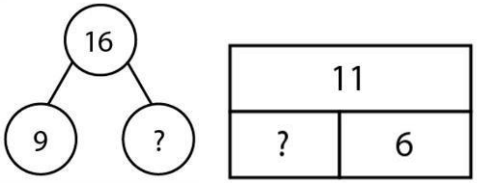
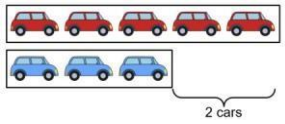
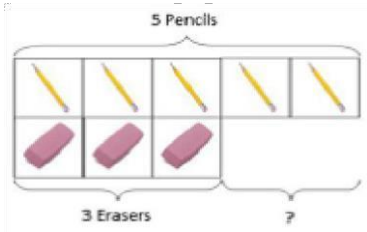
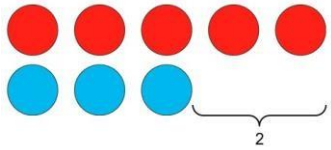
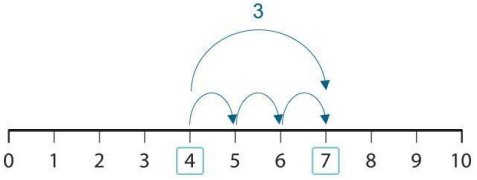
Year 3 Addition

Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
	When moving from concrete to pictorial, show concrete alongside pictorial. Show pictorial alongside abstract when moving to abstract.		
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3digit numbers.  4 3 + 2 5 _____	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame.  3 7 9	248 + 131 _____ 379 _____ Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping. Use language of *'take and make'* to describe carrying	 Exchange ten ones for a ten. Model using numicon and pv counters. 	Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line 	Use expanded method ONLYWHEN NEEDED 20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 11

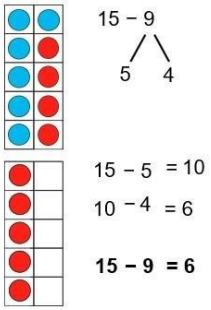
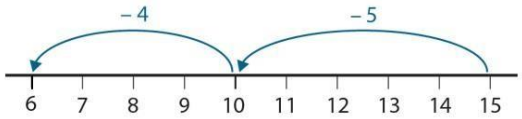
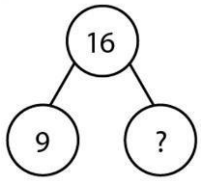
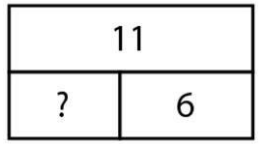
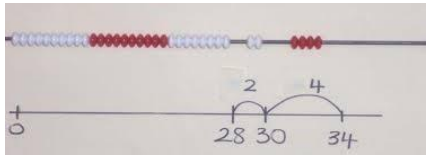
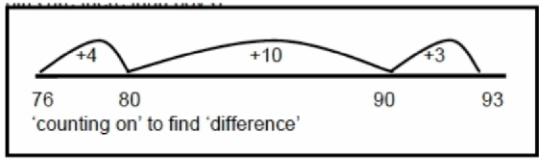
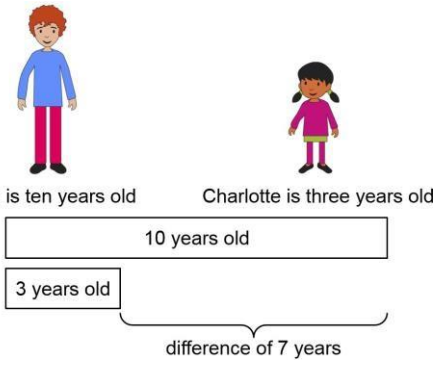
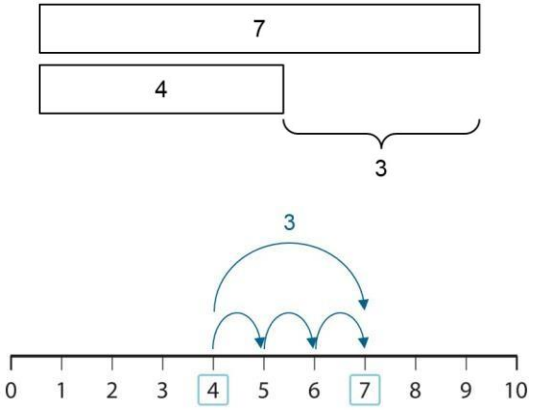
Year 4-6 Addition

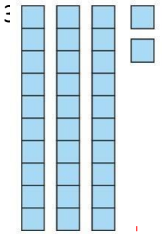
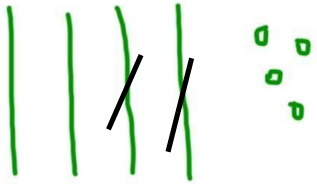
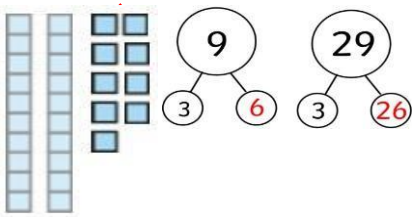
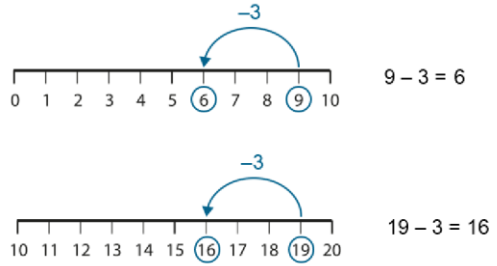
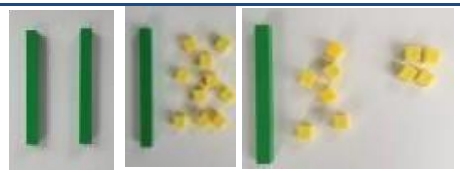
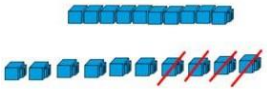
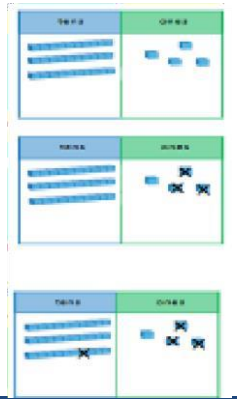
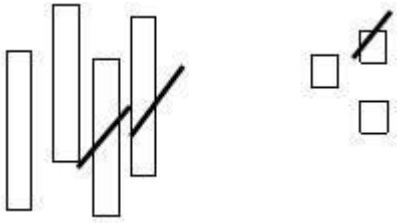
Objective ,Strategy Key Vocabulary	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	2634 + 4517 7141 1 1 Continue from previous work to carry ones, tens and hundreds. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters		22,634 + 15,673 38,307 1 1 £127.67 +£ 38.45 £166·12 1 1 1
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	Some children may need to ruse manipulatives and/or representations for longer. See year 5		89,472 63,673 + 3,016 156,161 1 1 1 1 Insert zeros for place holders. 1.437 0.600 +3.020 4·057 1

Year 1 Subtraction

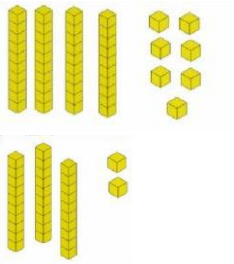
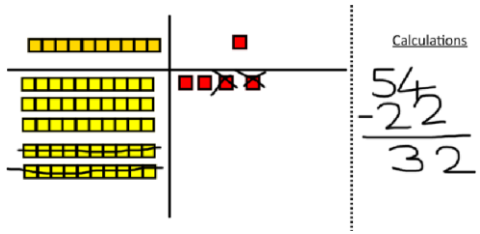
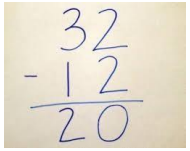
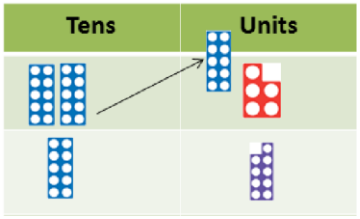
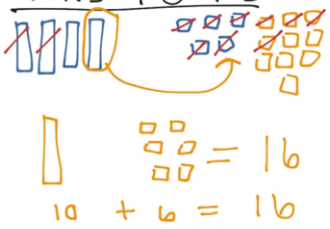
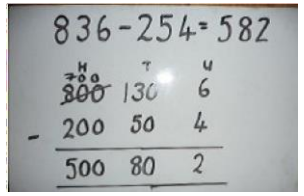
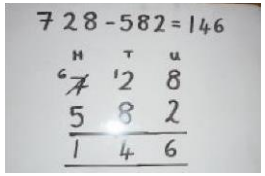
Objective, Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part-Part-Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model.  12 - 5 = 7 12 - 7 = 5 7 = 12 - 5 5 = 12 - 7
Subtract by making ten	15—9 Make 15 on the ten frame. Take 5 away to make ten, then take 4 more away so that you have taken 9. 	15 – 9  Jump back 5 first, then another 4. Use ten as the stopping point.	16—9 How many do we take off first to get to 10? How many left to take off? 
Compare numbers by finding the difference.	 There are 2 more red cars than blue cars.  There are 2 more pencils than erasers.	 5 – 3 = 2 Use a number line to count on.. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister?

Year 2 Subtraction

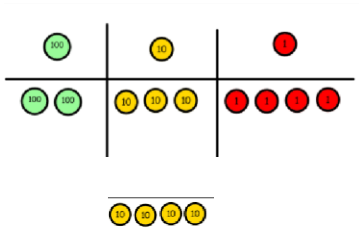
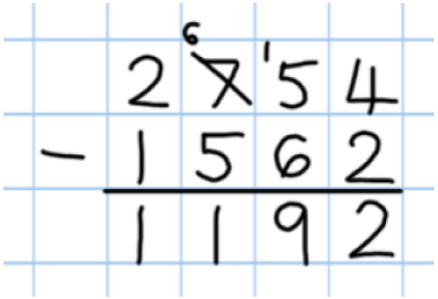
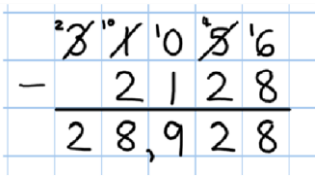
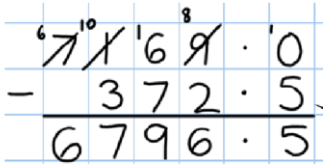
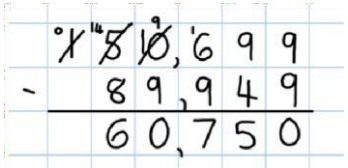
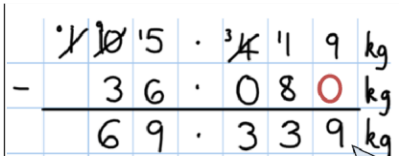
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting by making 10	**15 - 9 =.** Make 15 on the ten frame. Take 5 away to make ten, then take 4 more away so that you have taken 9.  	15 - 9 =  Jump back 5 first, then another 4. Use ten as the stopping point.	16 - 9 = How many do we take off first to get to 10? How many left to take off?  
Counting on to next ten *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	34 - 28 =  34—28 Use a bead bar or bead strings to model counting to next ten and the rest. 28 to 30 is 2, 30 to 34 is 4. So, 34 - 28 = 6	 Use a number line to count on to next ten and then the rest. Begin with bead line, move to landmarked line then to ENL.	93—76 = 17 76 → 80 = 4 80 → 93 = 13 13 + 4 = 17
Subtractions as difference	 	 	The difference between 24 and 16 is 8.

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting a multiple of 10	 32 - 10 = 22 Children use dienes, PV counters or Numicon. They remove the correct number of tens	 Children draw rods and cu-bes and cross off multiples of ten.	64 - 10 = ☐ 64 - 20 = ☐ 64 - 30 = ☐ 64 - ☐ = 24 ☐ - 50 = 14
Subtract a single digit from a two digit number No regrouping		 9 – 3 = 6 19 – 3 = 16	9 - 3 = 6 19 - 6 = 13 29 - 6 = 23 etc
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'.	20 - 4 = 16 	20— 4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21 Use Dienes to show how to partition the number when subtracting without regrouping. 	43—21 = 22 Children draw representations of Dienes and cross off. 	43—21 = 22

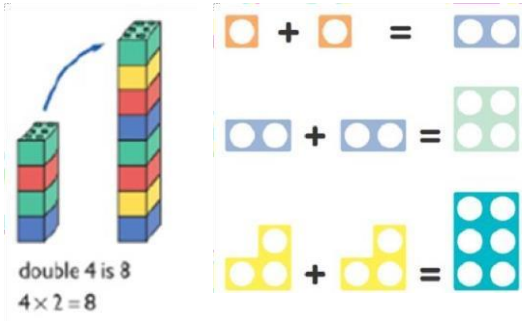
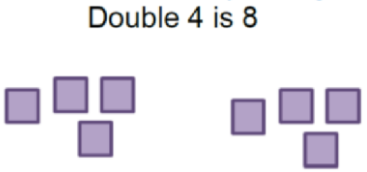
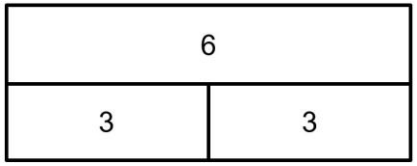
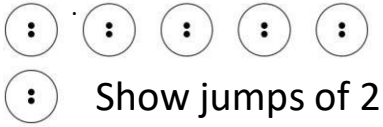
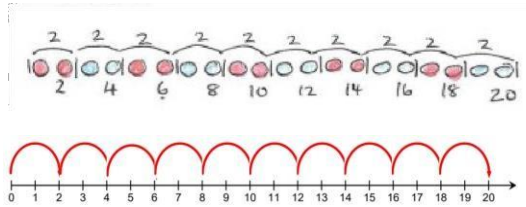
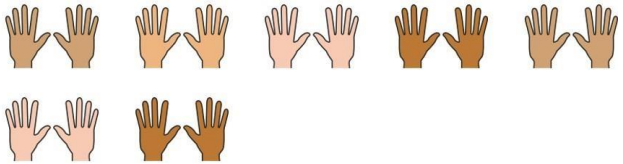
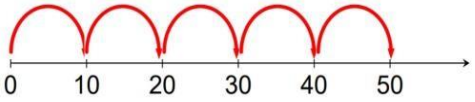
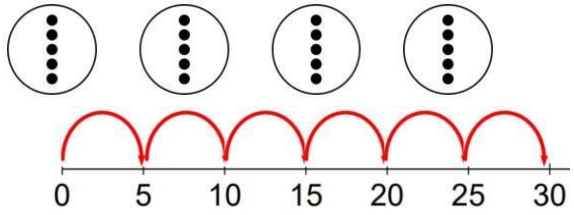
Year 3 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 47—32 — Use base 10 or Numicon to model	 Draw representations to support understanding	$47-24=23$ $40+7$ $-\ 20+4$ $20+3$ Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	45 -29 16  Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

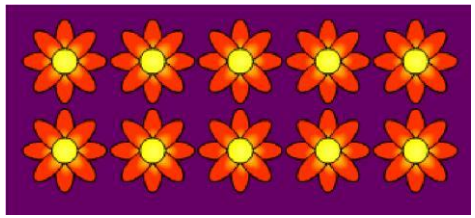
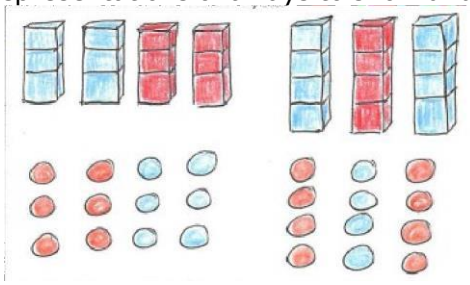
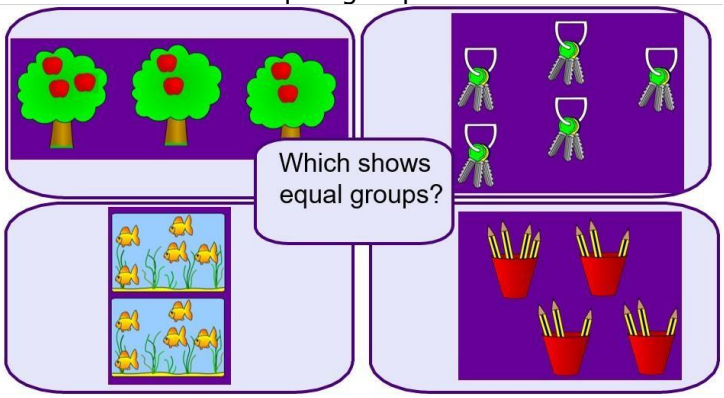
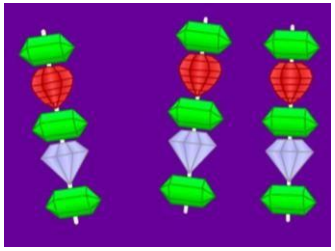
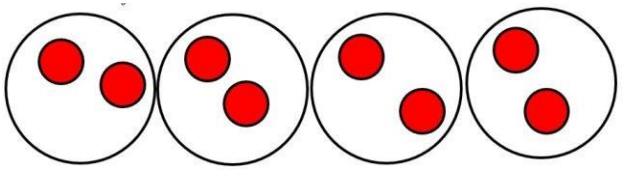
Year 4-6 Subtraction

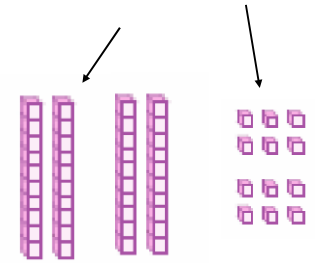
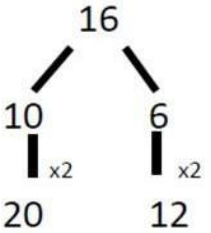
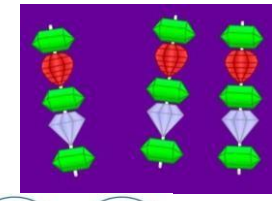
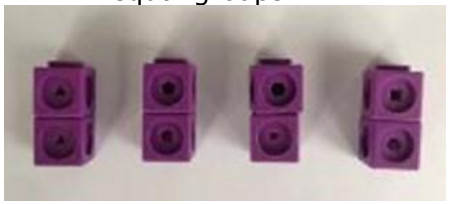
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point.*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

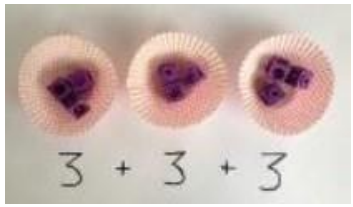
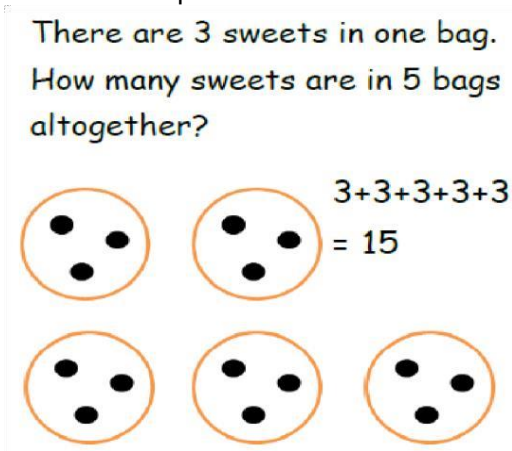
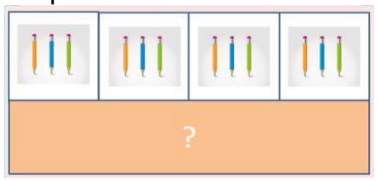
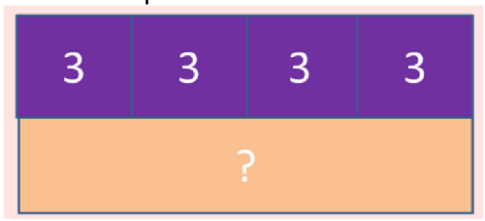
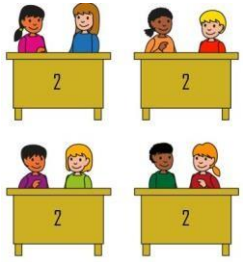
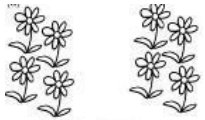
Year 1 Multiplication

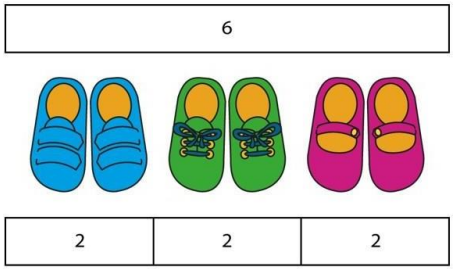
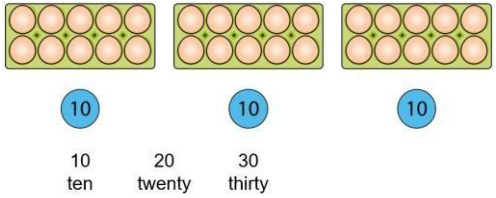
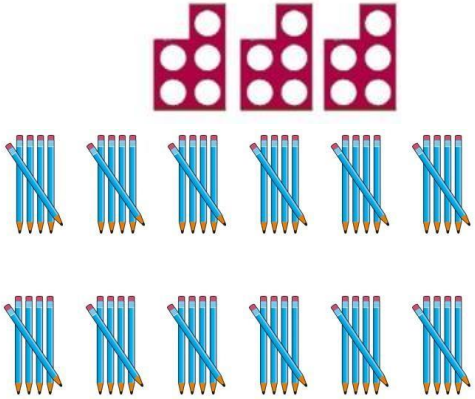
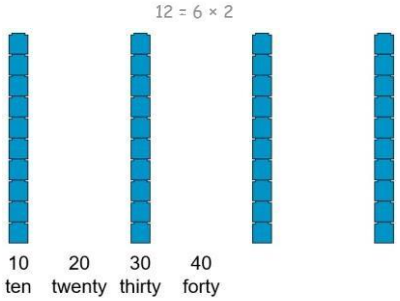
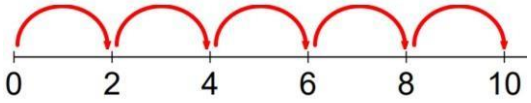
Objective & Strategy	Concrete	Pictorial
Double numbers to 10	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures and bar models to show how to double numbers  
Counting in groups of 2	Count in 2s using real life objects and contexts. 	Children make representations to show counting in multiples of 2. Count in multiples of a number aloud.  Show jumps of 2 on number line 
Counting in groups of 10	Use real life objects and contexts to count in groups of 10 	Use and draw representations for counting in multiples of 10. Count in multiples of 10 aloud Show jumps of 10 on a number line  
Counting in groups of 5	Use real life objects and contexts to count in groups of 5 	Use and draw representations for counting in multiples of 5. Count in 5s aloud. 

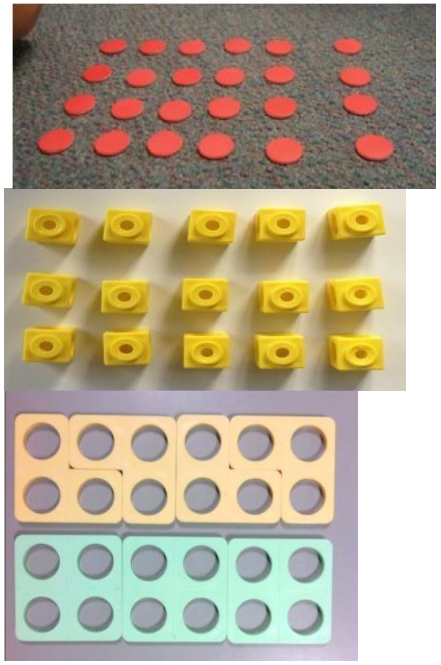
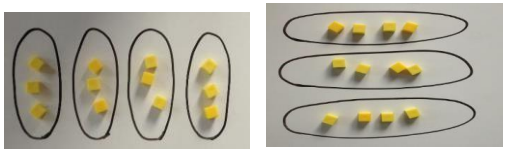
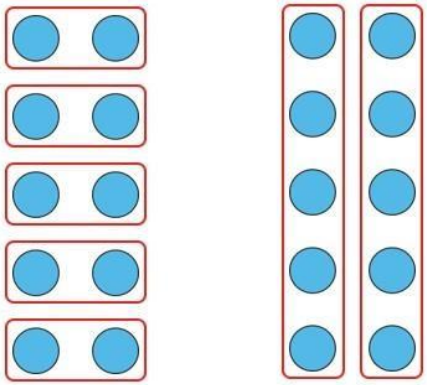
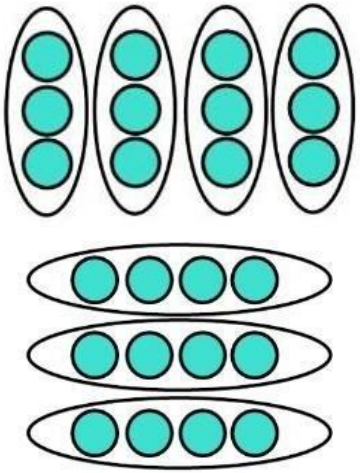
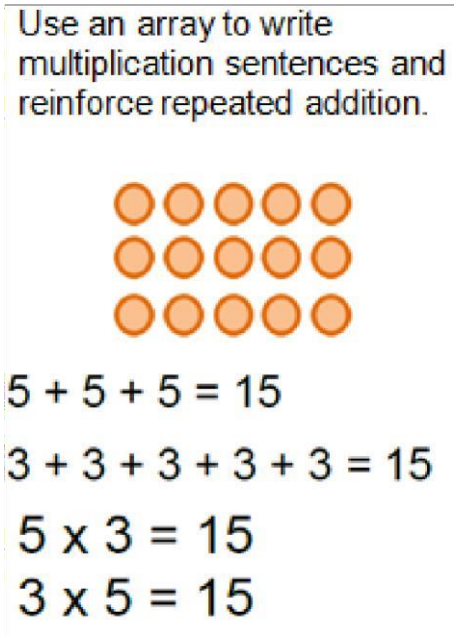
Year 2 Multiplication

Objective & Strategy	Concrete	Pictorial
Understand and use arrays	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 etc. 	Make and draw representations of arrays to show understanding 
Equal/non equal groups	Use real life objects and contexts to examine equal and non-equal groups.   There are 3 equal groups. There are 5 in each group.	Children make/match representations of real life problems to show equal groups and find the total.  There are 4 equal groups. There are 2 in each group. There are 8 altogether.

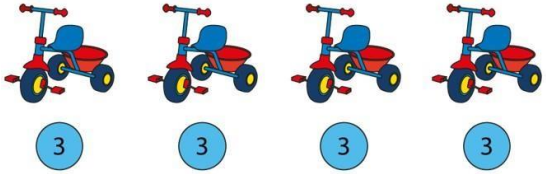
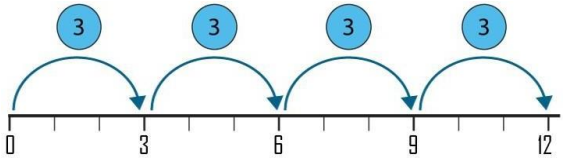
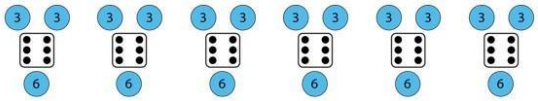
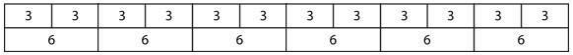
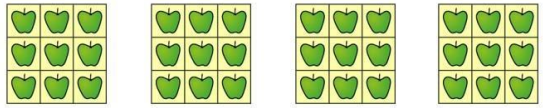
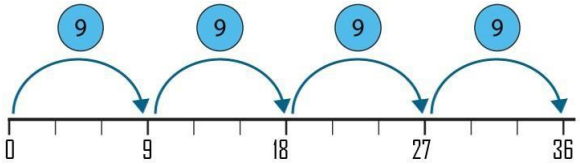
Objective & Strategy	Concrete	Pictorial	Abstract
Double a 2-digit number	Model doubling using dienes and PV counters. 40 + 12 = 52 	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  + = 32
Understand equal and non-equal groups	These are non– equal groups  These are equal groups   There are 5 equal groups. Each group has 3 cakes.	Make representations and drawings of equal groups   I have 4 groups of 3.	

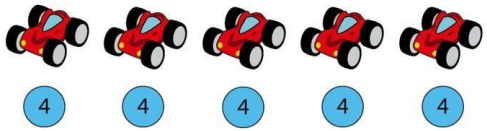
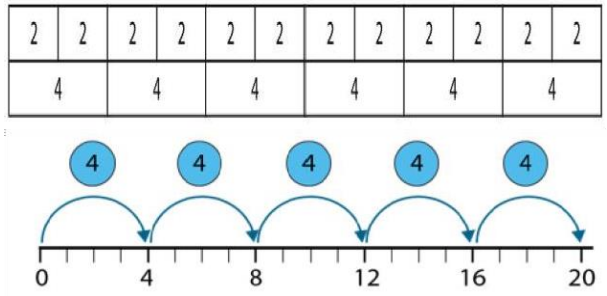
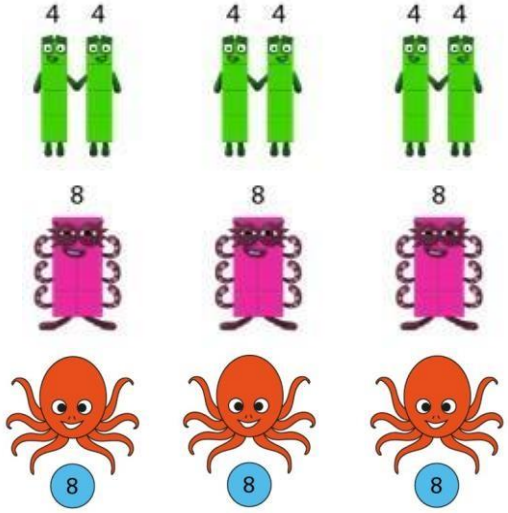
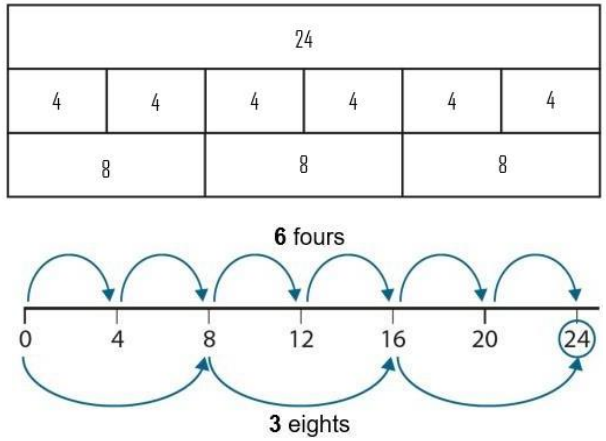
Objective & Strategy	Concrete	Pictorial	Abstract
Use repeated addition for multiplications	Use objects and real life contexts.  There are 5 groups of 2. There are 10 socks altogether.  There are 3 groups of 3. There are 9 altogether.	Make and draw representations to show repeated addition  Use bar models for representations of repeated additions.  	Create number sentences using repeated addition to match representations.  3 + 3 + 3 + 3 = 12
Relate repeated addition to multiplication using the x sign.	Write multiplication sentences to match repeated addition.  2 + 2 + 2 + 2 4 × 2	Children make and draw representations  and record both an addition sentence and a multiplication sentence. 1 + 1 + 1 + 1  6 x 1 + 6 □ x □ = 8	Write multiplication sentences to match repeated addition, without the support of representations. 2 + 2 + 2 + 2 + 2 = 10 5 x 2 = 10

Objective & Strategy	Concrete	Pictorial	Abstract
Understand the 2, 5 and 10 times table	Use objects and real life contexts for multiples of 2, 5 and 10  3 × 2 = 6 6 = 3 × 2  3 × 10 = 30 	Make and draw representations for 2, 5 and 10 times tables   4 × 10 = 40 Number lines, bead strings, counting sticks and bar models should be used to show representation of counting in multiples.  5 x 2 = 10 	Understand the terms factor and product 3 × 2 = 6 factor × factor = product 6 = 3 × 2 product = factor × factor Count in multiples of a number aloud.

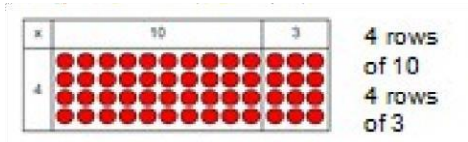
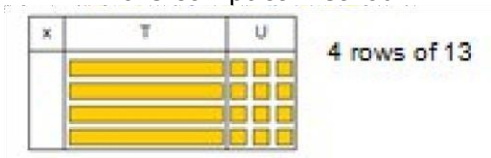
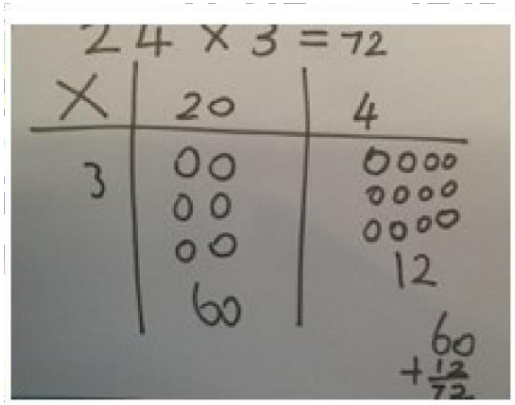
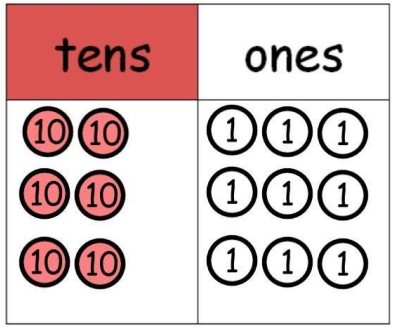
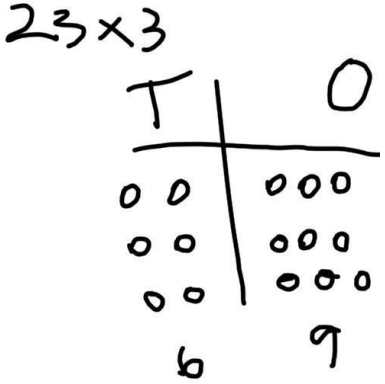
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity.  5 × 2 = 10 5 groups of 2 2, five times 5 × 2 = 10 2 groups of 5 5, two times 	12 = 3 × 4 12 = 4 × 3  Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Year 3 Multiplication

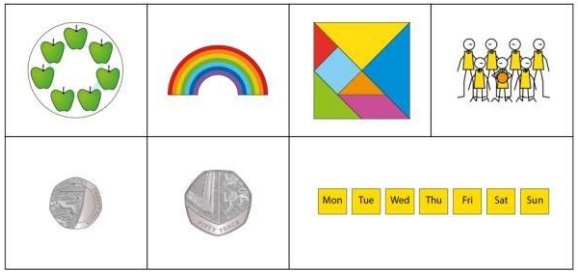
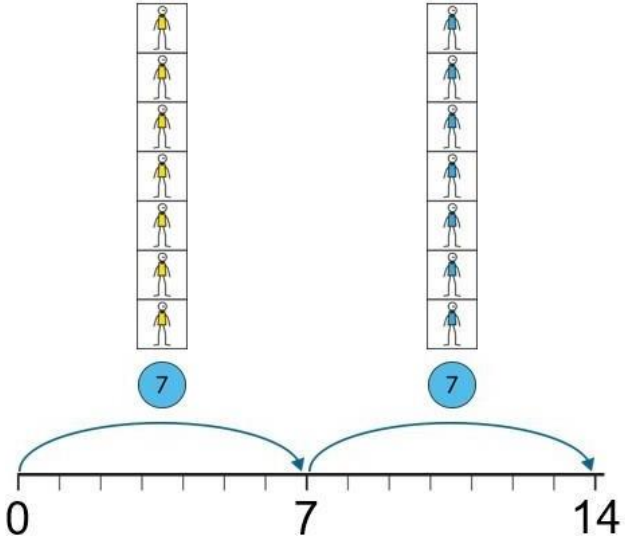
Objective & Strategy	Concrete	Pictorial	Abstract
Understand the 3 times table	Count in three using objects and representations of multiples of 3. 		There are 12 wheels. $4 \times 3 = 12$ $3 \times 4 = 12$
Understand the 6 times table	We can double our 3 times table to find our 6 times table. 		12 x 3 = 36 6 x 6 = 36
Understand the 9 times table	Count in nines using objects and representations of multiples of 9. Make links 9 being three groups of three. 		There are 36 apples. $4 \times 9 = 36$ $9 \times 4 = 36$

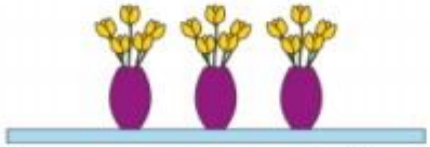
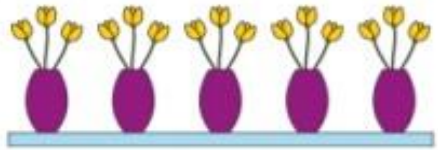
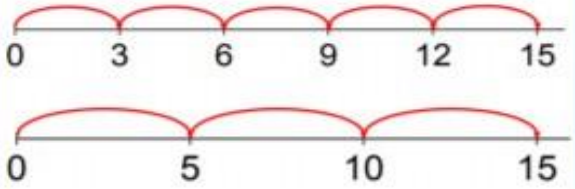
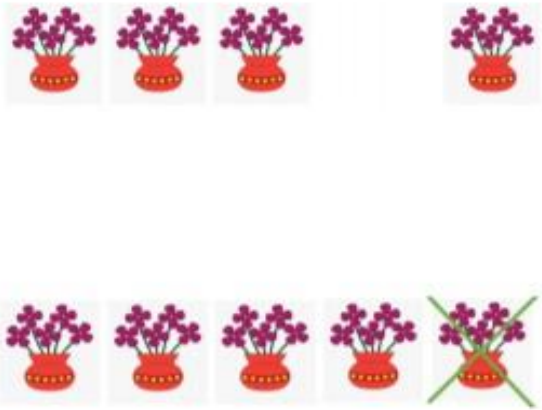
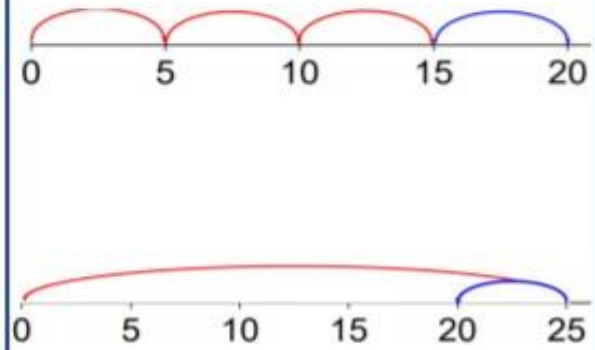
Objective & Strategy	Concrete	Pictorial	Abstract
Understand the 4 times table	We can double our 2 times table to get the 4 times table  Pupils revise 2 times table from year 2 and make link that this can be doubled to obtain 4 times table. How many wheels? Count in groups of 4. 	 	12 x 2 = 24 6 x 2 = 24 There are 20 wheels. $5 \times 4 = 20$ $4 \times 5 = 20$
Understand the 8 times table	We can double our 4 times table to get the 8 times table 	 	6 x 4 = 24 3 x 8 = 24

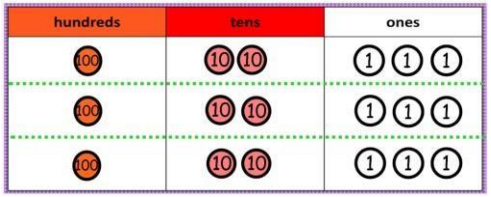
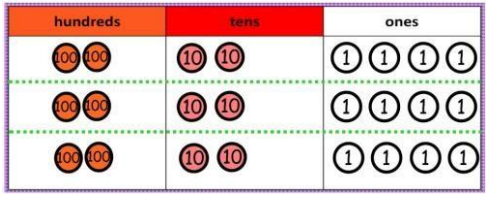
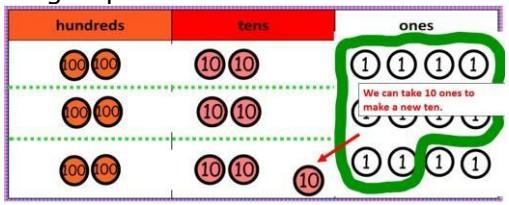
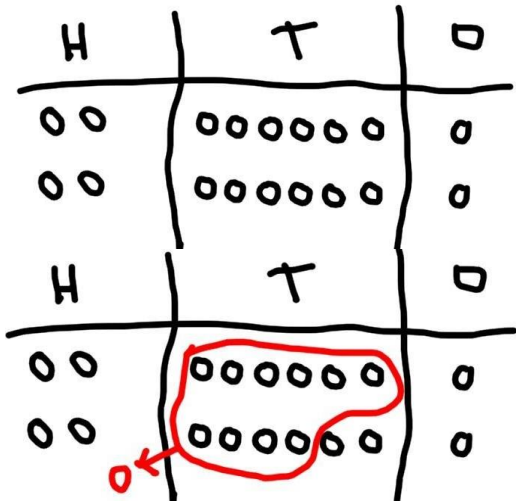
Divisibility rules in 'families' – 2, 4 and 8	
2	A number is divisible by 2 if the ones digit is even.
4	If halving a number gives an even value, then the number is divisible by 4. *and* For numbers with more than two digits: if the final two digits are divisible by 4 then the number is divisible by 4.
8	If halving a number twice gives an even value, the number is divisible by 8.

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying 2-digit by 1 digit using partitioning (distributive law)	 Show the links with arrays to illustrate the PV partitioning Move onto base ten to move towards a more compact method.  Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	4 x 10 = 40 4 x 3 = 12 40 + 12 = 52
2 digit x 1 digit using PV counters (no regrouping)	23 x 3  Chn can see array in the ones and the tens. There is a visual link to repeated addition.	Children practice, drawing their representations. 	$\begin{array}{r} 23 \\ \times \quad 3 \\ \hline 69 \end{array}$

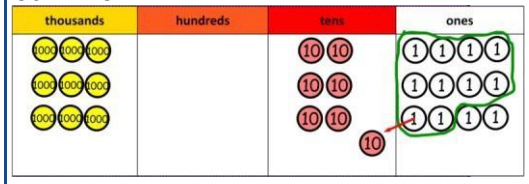
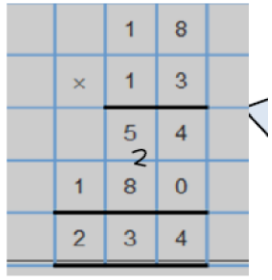
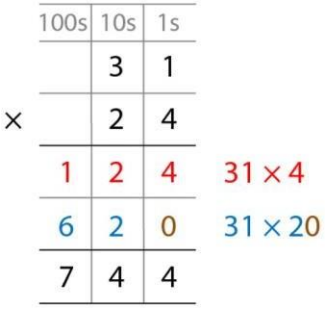
Year 4 Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Understand the 7 times table	Children use representations which show  groups of 7 including real life contexts.	Linear models show jumps of 7. 	There are 14 players. $2 \times 7 = 14$ $7 \times 2 = 14$

Objective & Strategy	Concrete	Pictorial	Abstract
Understanding the commutative law.	A  • *'Three groups of five are equal to fifteen.'* • *'Five, three times is equal to fifteen.'* B  • *'Five groups of three are equal to fifteen.'* "Three groups of five is equal to five groups of three."	 	3 x 5 = 15 5 x 3 = 15 5 x 3 = 3 x 5 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3
Understanding the distributive law		 	4 x 5 = 3 x 5 + 5 = 20 4 x 5 = 5 x 5 – 5 = 20

Objective & Strategy	Concrete	Pictorial	Abstract
Multiply 3 digit numbers by 1 digit. (no exchange)	Use place value counters to show how we are finding groups of a number. We are multiplying by 3 so we need 3 rows 123 x3 = 369  300 + 60 + 9 Add up each column, starting with the ones.	Children can represent their work with place value counters by drawing place value counters or Dienes.	231 x 3 693 3 x 1 *ones* is three ones 3 x 3 *tens* is nine tens 3 x 2 *hundreds* is six hundreds
Multiply 3 digit numbers by 1 digit. (with exchange)	224 x 3  Regroup ten ones to make a new ten.  600 + 70 + 2 =672	261 x 2  500 + 20 + 2 =522	241 x 4 964 1 4 times 1 *ones is* 4 ones 4 times 4 *tens is* 16 tens. I put 6 tens down and carry ten tens which is now a hundred. 4 times 2 *hundreds is* 8 hundreds. I add the hundred I have carried to make 9 hundreds.

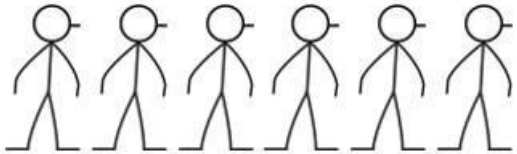
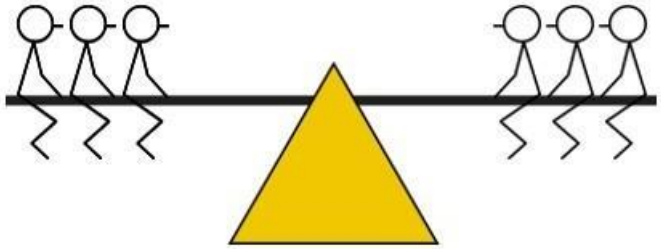
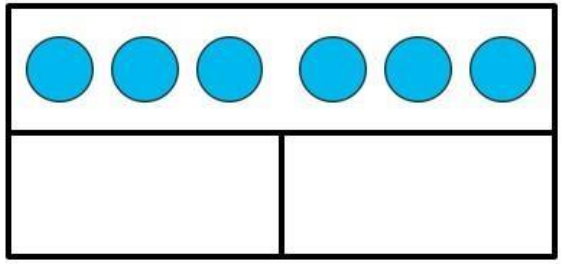
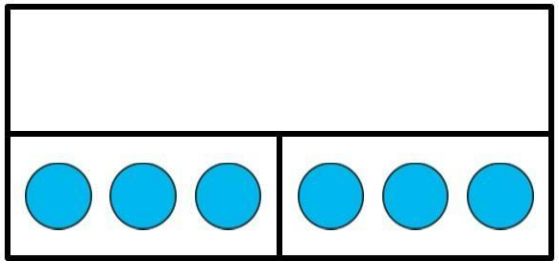
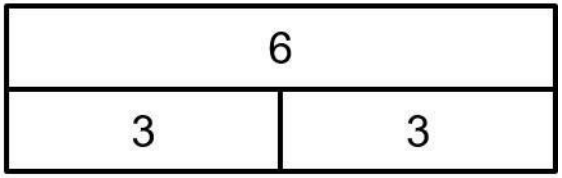
Year 5 Multiplication

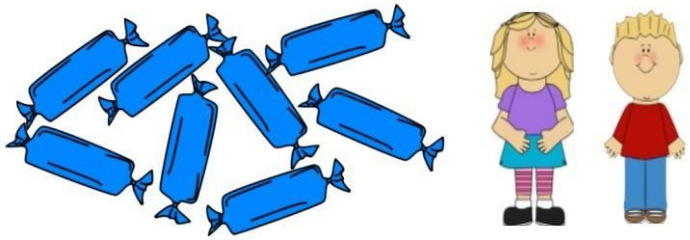
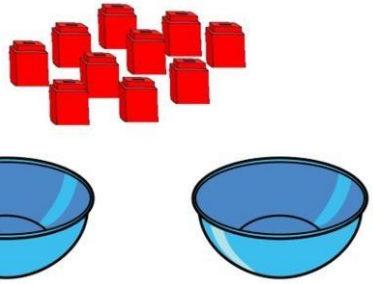
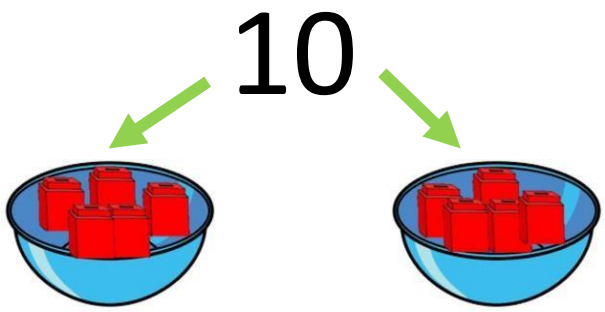
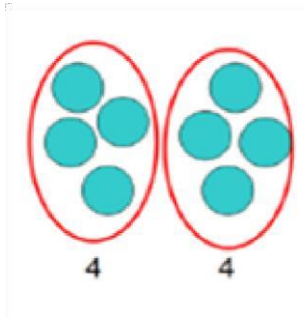
Objective & Strategy	Concrete	Pictorial	Abstract
Multiply 3 and 4 digits x 1 digit.	Children may continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 3024 x 3  900 + 0 + 60 +	Children may continue to draw their understanding using place value grids.	3 0 2 4 x 3 9 0 7 2 1
Multiply up to 4 digits by 2 digits	Manipulatives may still be used with the corresponding long multiplication modelled alongside. Begin with teen number x teen number. Progress to any 2 –4 digit number x 2 digit.	   	1 8 × 1 3 5 4 2 1 8 0 2 3 4 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first

Year 6 Multiplication

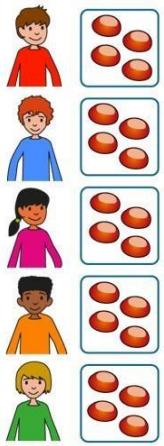
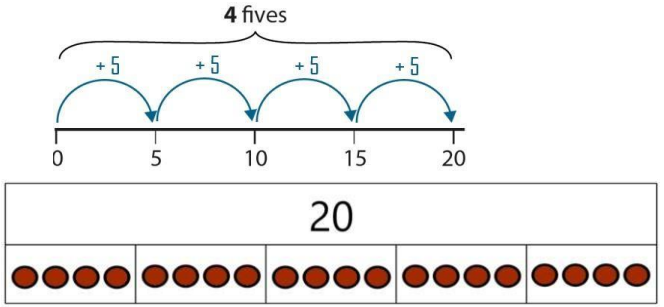
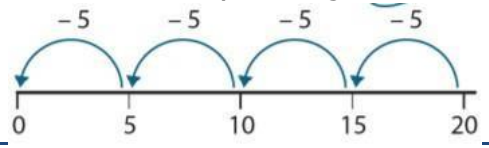
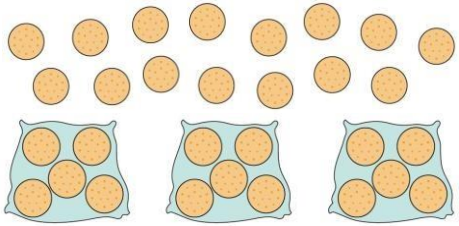
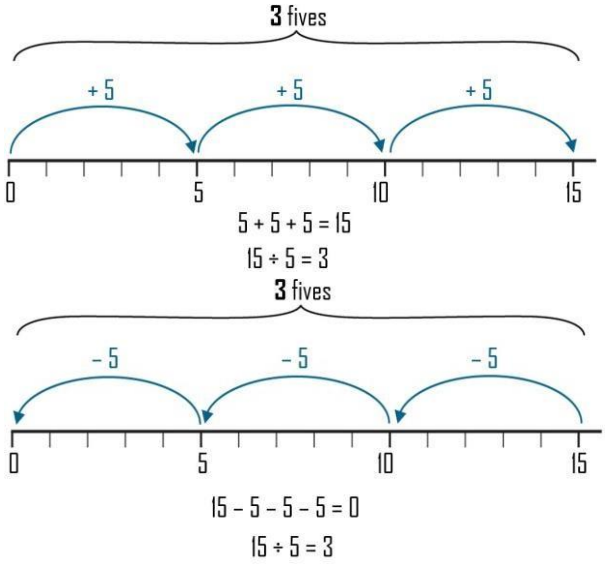
Objective & Strategy	Concrete	Pictorial	Abstract
Multiply decimals up to2 decimal places by a single digit			2.38 x 3 = 7.14 (1 2) First we lay out the calculation Next, we write the decimal point in the answer (product). Finally, we carry out the multiplication. 3 x 8 *hundredths* is 24 *hundredths* 3 x 3 *tenths* is 9 *tenths*, add 2 *tenths* we carried is *11 tenths* 3 x 3 *ones* is 6 *ones*, add 1 *one* we carried is 7 *ones*
Multiply up to 4 digit numbers by 2 digits.			312 × 28 = 2496 + 6240 = 8736 (carry: 1)

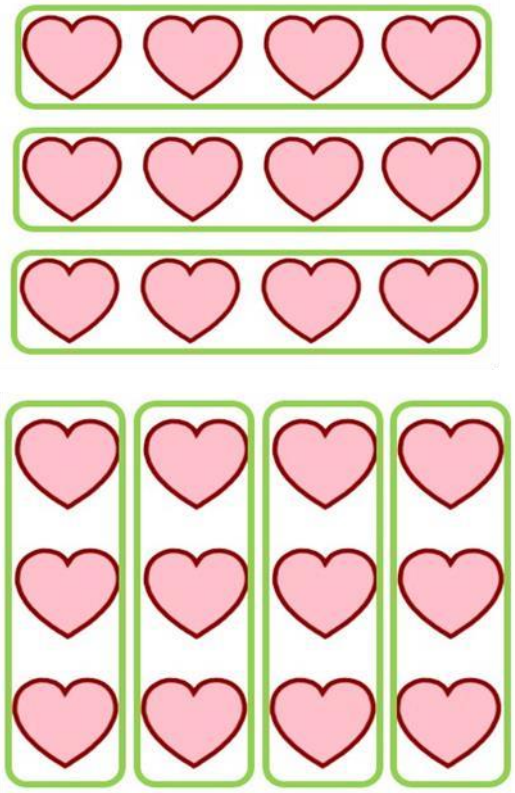
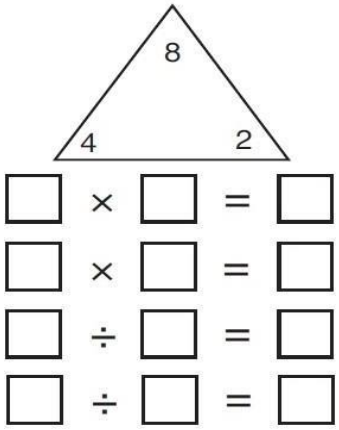
Year 1 Division

Objective & Strategy	Concrete	Pictorial
Find half of numbers to 20.	Real ife and practical contexts are used to find half of numbers up to 20.   	Children use manipulatives to represent real life problems.      half of 6 = 3 double 3 = 6

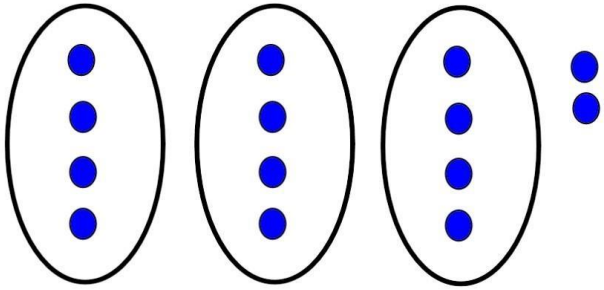
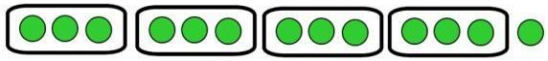
Objective & Strategy	Concrete	Pictorial
Understand division as sharing into equal groups *Use White Rose ITPs for modelling*	 Children solve real life problems using real objects. There are eight sweets. Daisy and Will share these equally. How many do they get each? I have 10 cubes, can you share them equally in 2 groups?  There are 2 equal groups. Each group has 5. 	Children use pictures or shapes to share quantities. 8 shared between 2 is 4 

Year 2 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing (partitive)	 There are 20 conkers shared equally between 5 children. Each child gets 4 conkers.	Children use pictures or shapes to share quantities. They may use bar modelling to show and support understanding.  Number lines are used to show skip counting (counting forwards) and repeated subtraction (counting backwards). 	20 ÷ 5 = 4
Division as grouping (quotitive)	Use cubes, counters or real objects or to aid understanding. There are 15 biscuits, there are 5 in each bag. How many bags? 		15 divided into groups of 5 is 3 15 ÷ 5 = 3

Objective & strategy	Concrete	Pictorial	Abstract
Understanding the inverse			3 x 4 = 12 12 ÷ 4 = 3 4 x 3 = 12 12 ÷ 3 = 4 2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.

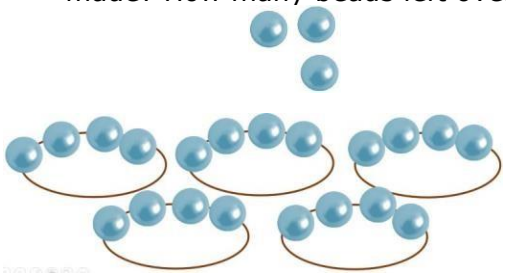
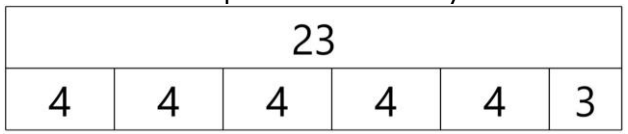
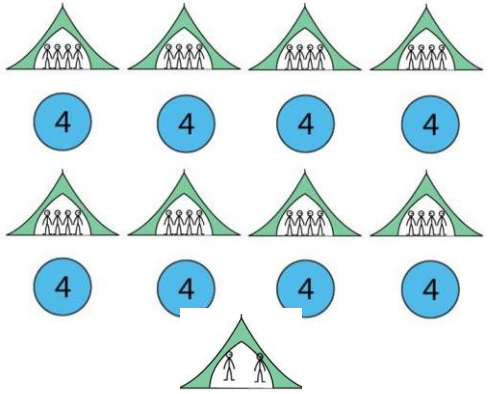
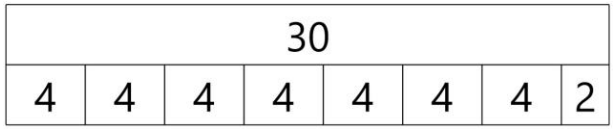
Year 3 Division

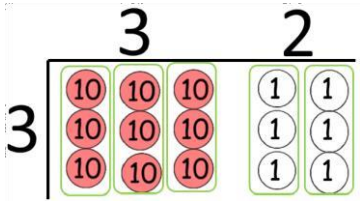
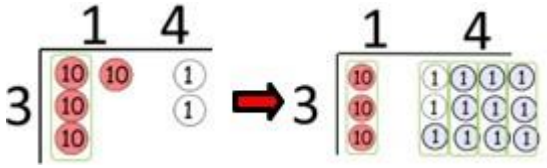
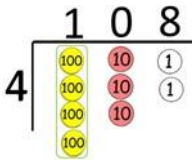
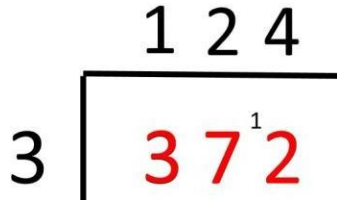
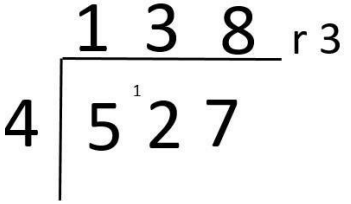
Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders. (partitive)	I divide 14 cakes between 3 plates. How are the cakes shared? 	Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 14 ÷ 3 = 4 r 2 dividend divisor quotient remainder
Division with remainders. (quotitive)	13 eggs are put into boxes. Each box holds 3 eggs. How are the eggs boxed? 	Children may draw representations to show their understanding.  Use bar models to show division with remainders. 13 3 3 3 3 1	13 ÷ 3 = 4 r 1

Divisibility rules in 'families' – 3, 6 and 9	
3	For a number to be divisible by 3, the sum of the digits of the number must be divisible by 3.
6	For a number to be divisible by 6, the number must be divisible by *both* 2 *and* 3.
9	For a number to be divisible by 9, the sum of the digits of the number must be divisible by 9.

Divisibility rules in 'families' – 5 and 10	
5	A number is divisible by 5 if the ones digit is 5 or 0.
10	A number is divisible by 10 if the ones digit is 0.

Year 4 Division

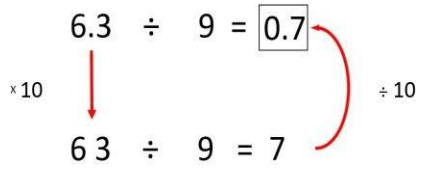
Objective & Strategy	Concrete	Pictorial	Abstract
Interpreting division with remainders.	Bracelets are made using 4 beads. There are 23 beads. How many bracelets can be made? How many beads left over? 	Bar model representations may be used. 	23 ÷ 4 = 5 r 3
Interpreting division with remainders.	4 scouts can fit in each tent. How many tents needed for 30 scouts? 		30 ÷ 4 = 7 r 2 8 tents are needed. Discuss with pupils the need to round up in this context.

Objective & Strategy	Concrete	Pictorial	Abstract
Divide 2 & 3 digit numbers by 1 digit Short Division	96 ÷3 Use place value counters to make groups of the divisor, starting with the largest value digit.  There are 3 groups of 3 tens. There are 2 groups of 3 ones. 42 ÷ 3  There is 1 group of 3 tens. There is a ten left over. We exchange this for 10 ones. 12 ones divided by 3 is 4. 432 ÷ 4  There is 1 group of 4 hundreds. There are no groups of 4 tens and 3 tens left over. There are 8 groups of 4 ones after exchanging the left over tens.	Students use drawn diagrams with spots or circles to show their understanding.	Begin with divisions that divide equally with no remainder.  Move on to divisions with a remainder. Return to concrete if necessary. 

Divisibility rules in numerical order	
2	A number is divisible by 2 if the ones digit is even.
3	For a number to be divisible by 3, the sum of the digits of the number must be divisible by 3.
4	If halving a number gives an even value, then the number is divisible by 4. *and* For numbers with more than two digits: if the final two digits are divisible by 4 then the number is divisible by 4.
5	A number is divisible by 5 if the ones digit is 5 or 0.

Divisibility rules in numerical order	
6	For a number to be divisible by 6, the number must be divisible by *both* 2 *and* 3.
8	If halving a number twice gives an even value, the number is divisible by 8.
9	For a number to be divisible by 9, the sum of the digits of the number must be divisible by 9.
10	A number is divisible by 10 if the ones digit is 0.

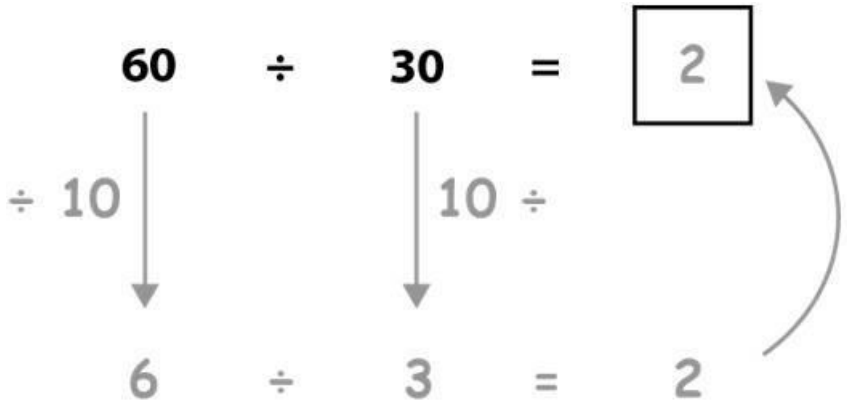
Year 5 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Divide decimals by a single digit, using x and ÷ by 10 or 100			Pupils understand the use of X and ÷ 10 to make connections. 
Short division of decimals			Children build on work from Year 4, now with decimals. 

Year 6 Division

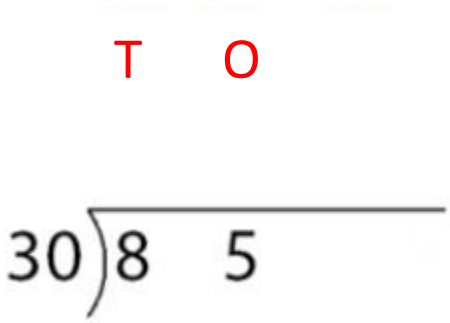
Division of 2 digits by 2 digits

Using x & ÷ by 10, 100 etc and relating this to a short division method.

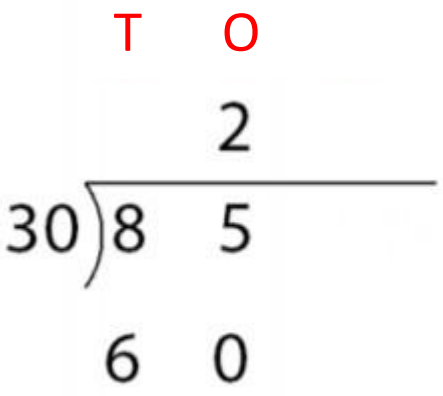


$$
\begin{array}{r}
\ 0\ \ 2 \\
30\overline{)\ 6\ \ {}^{6}0}
\end{array}
$$

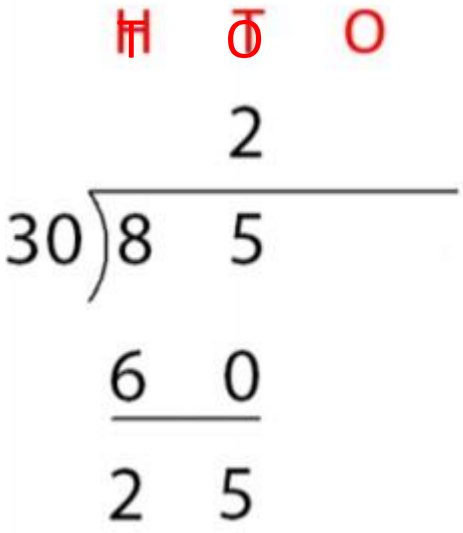
Long Division—2 digits divided by 2 digits



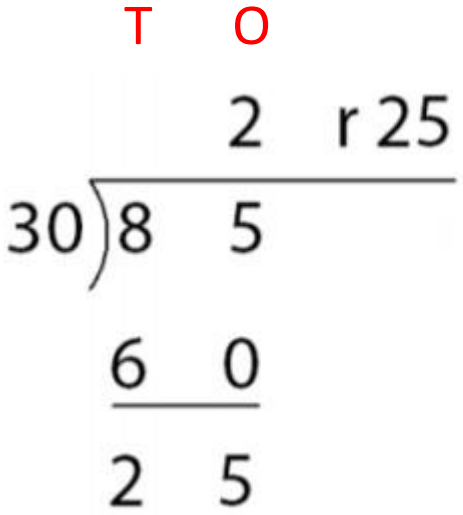
30 does not go into 8. So, combine the 8 tens with the 5 ones.



30 goes into 85 twice, which is 60.

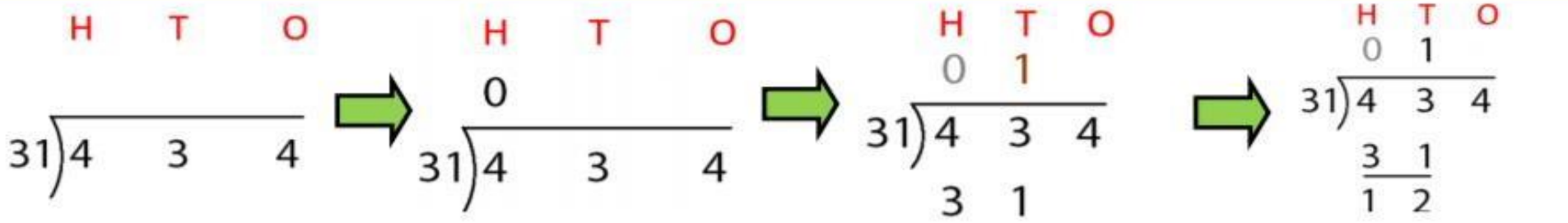


Subtract the 60 from the 85 and this leaves 25.



85 divided by 30 is 2 with a remainder of 25

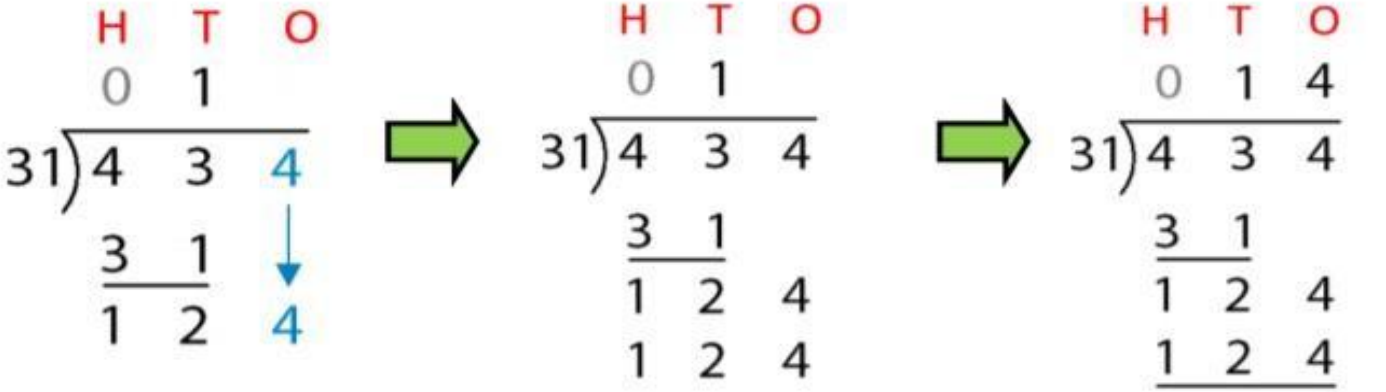
Long Division—3 digits divided by 2 digits



31 does not go into 4 (hundreds).

We combine the 4 hundreds with the tens to give 43 tens. 31 goes into 43 once which is 31, we record this underneath.

Subtract to find the remainder. 31 from 43 leaves 12.

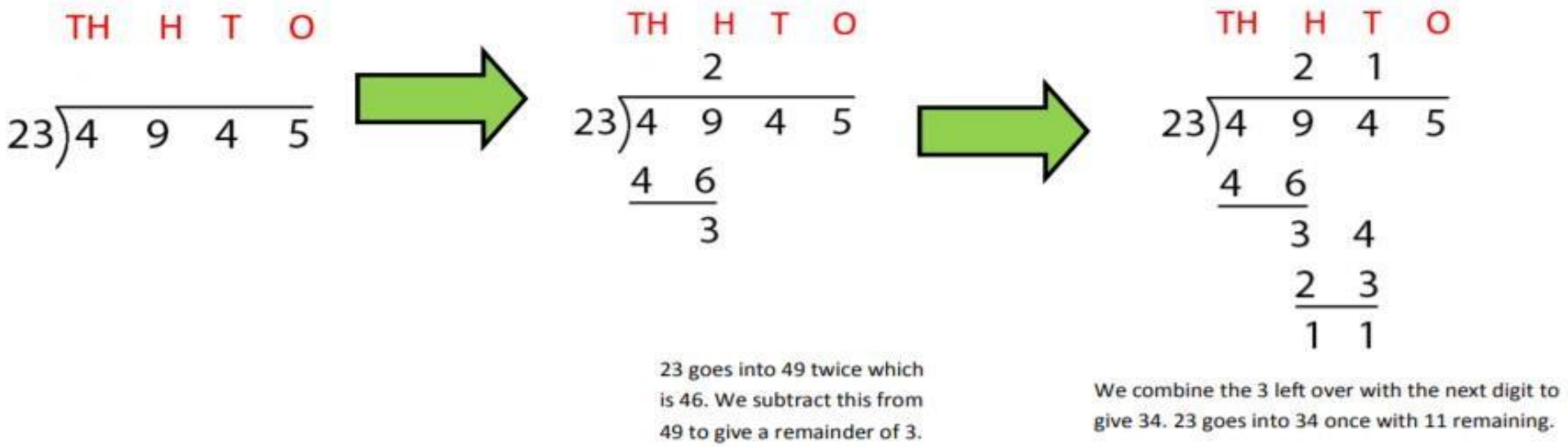


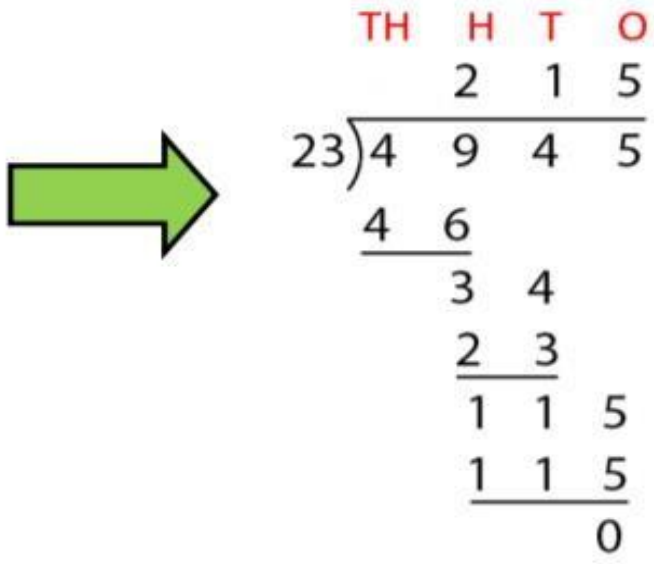
We combine 12 with the next digit to give 124.

31 goes into 124 four times, which is 124.

We subtract to show there is no remainder

Long Division—progressing to 4 or more digits





Long Division—procedural summary (remainder in the tens)

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 -4 1	t o 2 9 2)5 8 -4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 -4 1 8	t o 2 9 2)5 8 -4 1 8 -1 8 0	t o 2 9 2)5 8 -4 1 8 -1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Long Division—procedural summary (remainder in any of the digits)

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8	h t o 1 2)2 7 8 - 2 0	h t o 1 8 2)2 7 8 - 2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 - 2 0 7	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1	h t o 1 3 2)2 7 8 - 2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0	h t o 1 3 9 2)2 7 8 - 2 0 7 - 6 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.